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Original Article

2 – QUASI TOTAL SINGLE VALUED NEUTROSOPHIC GRAPH AND ITS PROPERTIES

G. DIVYA[1] & DR. J. MALARVIZHI[2]

[1]Research Scholar, PG and Research Department of Mathematics, K. N. Govt. Arts College (Autonomous) for women, Thanjavur – 7, Affliated to Bharathidasan University, Tamil Nadu, India

[2]Principal, Govt. Arts College, Ariyalur, Affliated to Bharathidasan University, Tamil Nadu, India

ABSTRACT

In this paper we construct the 2 – Quasi Total Single valued Neutrosophic Graph of the given Single valued Neutrosophic Graph. Some properties and relationships are observed. Isomorphic Relation between some special graphs are also discussed.

KEYWORDS: Quasi Total Single valued Neutrosophic Graph of the given Single valued Neutrosophic Graph

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1. INTRODUCTION

Fuzzy set theory and intuitionistic fuzzy sets theory are useful models for dealing with uncertainty and incomplete information. But they may not be sufficient in modeling of indeterminate and inconsistent information encountered in real world. In order to cope with this issue, neutrosophic set theory was proposed by Smarandache as a generalization of fuzzy sets and intuitionistic fuzzy sets.

Neutrosophic set is a powerful tool to deal with incomplete, indeterminate and inconsistent information in real world. It is a generalization of the theory of fuzzy set, intuitionistic fuzzy sets, interval-valued fuzzy sets and interval-valued intuitionistic fuzzy sets, then the neutrosophic set is characterized by a truth-membership degree (T), an indeterminacy-membership degree (I) and a falsity-membership degree (F)independently, which are within the real standard or nonstandard unit interval $]^{-}0, 1^{+}[$.

Properties and isomorphism of total and middle fuzzy graphs was given by Nagoorgani and Malarvizhi. Here, in this paper some properties of 1 – Quasi total Single valued Neutrosophic graphs is defined and isomorphic relation is discussed.

2. PRELIMINARIES

A Single-Valued Neutrosophic graph(SVN graph) is a pair G = (A,B) of the crisp graph G* = (V, E)(i.e., with underlying set V), where $A : V \to [0, 1]$ is single-valued neutrosophic set in V and $B : V \times V \to [0, 1]$ is single-valued neutrosophic relation on V such that

$$T_B(xy) \leq \min\{T_A(x), T_A(y)\},$$

$$I_B(xy) \leq \min\{I_A(x), I_A(y)\},$$

$$F_B(xy) \leq \max\{F_A(x), F_A(y)\}$$

for all x, y $\in$ V . A is called single-valued neutrosophic vertex set of G and B is called single-valued neutrosophic edge set of G, respectively.

Given a single-valued neutrosophic graph G = (A,B) of a crisp graph $G^* = (V, E)$, the order of G is defined as Order (G) = $(O_T(G), O_I(G), O_F(G))$, where $O_T(G) = \sum_{v\in V} T_A(v)$, $O_I(G) = \sum_{v\in V} I_A(v)$, $O_F(G) = \sum_{v\in V} F_A(v)$.

Given a single-valued neutrosophic graph G = (A,B) of a crisp graph $G^* = (V, E)$, the size of G is defined as Size(G) = $\left(S_T(G), S_I(G), S_F(G)\right)$, where $S_T(G) = \sum_{u\neq v} T_B(u, v)$, $S_I(G) = \sum_{u\neq v} I_B(u, v)$, $S_F(G) = \sum_{u\neq v} F_B(u, v)$.

The degree of a vertex x in an SVNG, G = (A, B) is defined to be sum of the weights of the edges incident at x. It is denoted by $d_G(u)$ and is equal to $(\sum_{u\neq v} T_B(u, v), \sum_{u\neq v} I_B(u, v), \sum_{u\neq v} F_B(u, v))$ for all v adjacent to u in G^*.

Two vertices x and y are said to be neighbors in SVNG if either one of the following conditions hold

- $T_B(x, y) > 0, I_B(x, y) > 0, F_B(x, y) > 0$
- $T_B(x, y) = 0, I_B(x, y) > 0, F_B(x, y) > 0$
- $T_B(x, y) > 0, I_B(x, y) = 0, F_B(x, y) > 0$
- $T_B(x, y) > 0, I_B(x, y) > 0, F_B(x, y) = 0$
- $T_B(x, y) = 0, I_B(x, y) = 0, F_B(x, y) > 0$
- $T_B(x, y) = 0, I_B(x, y) > 0, F_B(x, y) = 0$
- $T_B(x, y) > 0, I_B(x, y) = 0, F_B(x, y) = 0$ for x, y $\in$ A

Let G and G' be single valued neutrosophic graphs with underlying sets V and V' respectively. A homomorphism of single valued neutrosophic graphs, $h : G \to G'$ is a map $h : V \to V'$ which satisfies

$T_A(u) \leq T_{A'}(h(u)), I_A(u) \leq I_{A'}(h(u)), F_A(u) \leq F_{A'}(h(u))$ for all u $\in$ V

$T_B(u, v) \leq T_{A'}(h(u), h(v)), I_B(u, v) \leq I_{B'}(h(u), h(v)), F_B(u, v) \leq F_{B'}(h(u), h(v))$ for all $u, v \in V$.

Let G and G' be single valued neutrosophic graphs with underlying sets V and V' respectively. An isomorphism of single valued neutrosophic graphs, $h : G \to G'$ is a bijective map $h : V \to V'$ which satisfies

$T_A(u) = T_{A'}(h(u)), I_A(u) = I_{A'}(h(u)), F_A(u) = F_{A'}(h(u))$ for all u $\in$ V

$T_B(u, v) = T_{B'}(h(u), h(v)), I_B(u, v) = I_{B'}(h(u), h(v)), F_B(u, v) = F_{B'}(h(u), h(v))$ for all $u, v \in V$.Then G is said to be isomorphic to G'.Two isomorphic graphs are given below

A weak isomorphism of single valued neutrosophic graphs, $h : G \to G'$ is a map $h : V \to V'$ which is a bijective homomorphism that satisfies

$T_A(u) = T_{A'}(h(u)), I_A(u) = I_{A'}(h(u)), F_A(u) = F_{A'}(h(u))$ for all u $\in$ V

A co-weak isomorphism of single valued neutrosophic graphs, $h : G \to G'$ is a map $h : V \to V'$ which is a bijective homomorphism that satisfies

$T_B(u, v) = T_{B'}(h(u), h(v)), I_B(u, v) = I_{B'}(h(u), h(v)), F_B(u, v) = F_{B'}(h(u), h(v))$ for all $u, v \in V$.

The busy value of the vertex x in G is $BV(x) = (BV_{T_A}(x), BV_{I_A}(x), BV_{F_A}(x)) = (\sum_i T_A(x) \wedge T_A(x_i), \sum_i I_A(x) \wedge I_A(x_i), \sum_i F_A(x) \vee F_A(x_i))$ where x_i are the neighbours of x and the busy value of G is $BV(G) = \sum_i BV(x_i)$ where x_i are the vertices of G.

Let G: (A,B) be a SVN graph with the underlying crisp graph $G^* = (V, E)$. The vertices and edges of G are taken together as vertex set of $sd(G) = (A_{sd}, B_{sd})$, each edge 'e' in G is replaced by a new vertex and that vertex is made as a adjacent of those vertices which lie on 'e' in G. Here A_{sd} is a SVN subset defined on V ∪ E as

$(T_A, I_A, F_A)_{sd}(x) = (T_A, I_A, F_A)(x)$ if $x \in V$

$= (T_B, I_B, F_B)(x)$ if $x \in E$

The SVN relation B_{sd} on V ∪ E is defined as

$T_{B_{sd}}(x, y) = T_A(x) \wedge T_B(y)$ if $x \in V$ and $y \in E$

= 0 otherwise

$I_{B_{sd}}(x, y) = I_A(x) \wedge I_B(y)$ if $x \in V$ and $y \in E$

= 0 otherwise

$F_{B_{sd}}(x, y) = F_B(y)$ if $x \in V$ and $y \in E$

= 0 otherwise

$(T_{B_{sd}}, I_{B_{sd}}, F_{B_{sd}})(x, y)$ is a SVN relation on $(T_{A_{sd}}, I_{A_{sd}}, F_{A_{sd}})$ and hence the pair $sd(G) = (A_{sd}, B_{sd})$, is a SVN graph. This pair is said as subdivision SVN graph of G.

Let G=(A,B) be a SVN graph with its underlying crisp graph $G^* = (V, E)$. The pair $tl(G) = (A_{tl}, B_{tl})$ of G is defined as follows. The vertex set of tl(G) is V ∪ E. The SVN subset A_{tl} is defined on V ∪ E as,

$(T_A, I_A, F_A)_{tl}(x) = (T_A, I_A, F_A)(x)$ if $x \in V$

$= (T_B, I_B, F_B)(x)$ if $x \in E$

The SVN relation B_{tl} on V ∪ E is defined as

$T_{B_{tl}}(x, y) = T_B(x, y), I_{B_{tl}}(x, y) = I_B(x, y), F_{B_{tl}}(x, y) = F_B(x, y)$ if $(x, y) \in E$

$T_{B_{tl}}(x, y) = T_A(x) \wedge T_B(y)$ if $x \in V$ and $y \in E$

= 0 otherwise

$I_{B_{tl}}(x, y) = I_A(x) \wedge I_B(y)$ if $x \in V$ and $y \in E$

= 0 otherwise

$F_{B_{tl}}(x, y) = F_B(y)$ if $x \in V$ and $y \in E$

= 0 otherwise

$T_{B_{tl}}(e, f) = T_B(e) \wedge T_B(f)$ if $e, f \in E$ & they have a vertex in common

= 0 otherwise

$I_{B_{tl}}(e,f) = I_B(e) \wedge I_B(f)$ if $e, f \in E$ & $t$hey have a vertex in common

= 0 otherwise

$F_{B_{tl}}(e,f) = F_B(e) \vee F_B(f)$ if $e, f \in E$ & $t$hey have a vertex in common

= 0 otherwise

Thus by the definition B_{tl} is a single valued neutrosophic relation on A_{tl}. Hence the pair $tl(G) = (A_{tl}, B_{tl})$ is a SVN graph and is termed as Total Single Valued Neutrosophic Graph.

Let G=(A,B) be a SVN graph with its underlying crisp graph $G^* = (V, E)$. The vertices and edges of G are taken together as the vertex set of the pair M(G) = (A_M, B_M) where

$(T_A, I_A, F_A)_M(x) = (T_A, I_A, F_A)(x)$ if $x \in V$

$= (T_B, I_B, F_B)(x)$ if $x \in E$

$(T_B, I_B, F_B)_M(x,y) = 0$ if both $x, y \in V$

$T_{B_M}(e,f) = T_B(e) \wedge T_B(f)$ if $e, f \in E$ & *they have a vertex in common*

= 0 otherwise

$I_{B_M}(e,f) = I_B(e) \wedge I_B(f)$ if $e, f \in E$ & *they have a vertex in common*

= 0 otherwise

$F_{B_M}(e,f) = F_B(e) \vee F_B(f)$ if $e, f \in E$ & *they have a vertex in common*

= 0 otherwise

$T_{B_M}(x,y) = T_B(y)$ if $x \in V$ and $y \in E$

= 0 otherwise

$I_{B_M}(x,y) = I_B(y)$ if $x \in V$ and $y \in E$

= 0 otherwise

$F_{B_M}(x,y) = F_B(y)$ if $x \in V$ and $y \in E$

= 0 otherwise

As A_M is defined only through the values of A and B, $A_M: V \cup E \rightarrow [0,1]$ is well defined SVN subset on $V \cup E$. Also B_M is a SVN relation on A_M is also well defined. Hence the pair $M(G) = (A_M, B_M)$ is a SVN graph and is termed as Middle Single Valued Neutrosophic Graph.

Let G=(A,B) be a SVN graph with its underlying crisp graph $G^* = (V, E)$. The pair $Q_1 tl(G) = (A_{Q_1 tl}, B_{Q_1 tll})$ of G is defined as follows. The vertex set of $Q_1 tl(G)$ is $V \cup E$. The SVN subset A_{Q1tl} is defined on $V \cup E$ as,

$(T_A, I_A, F_A)_{Q_1 tl}(x) = (T_A, I_A, F_A)(x)$ if $x \in V$

$= (T_B, I_B, F_B)(x)$ if $x \in E$

The SVN relation B_{Q_1tl} on $V \cup E$ is defined as

$T_{B_{Q_1tl}}(x, y) = T_B(x, y), I_{B_{Q_1tll}}(x, y) = I_B(x, y), F_{B_{Q_1tl}}(x, y) = F_B(x, y)$ if $(x, y) \in E$

$T_{B_{Q_1tl}}(e, f) = T_B(e) \wedge T_B(f)$ if $e, f \in E$ & $t$hey have a vertex in common

$= 0$ otherwise

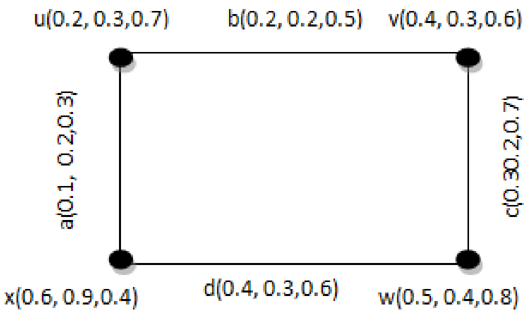
$I_{B_{Q_1tl}}(e, f) = I_B(e) \wedge I_B(f)$ if $e, f \in E$ & $t$hey have a vertex in common

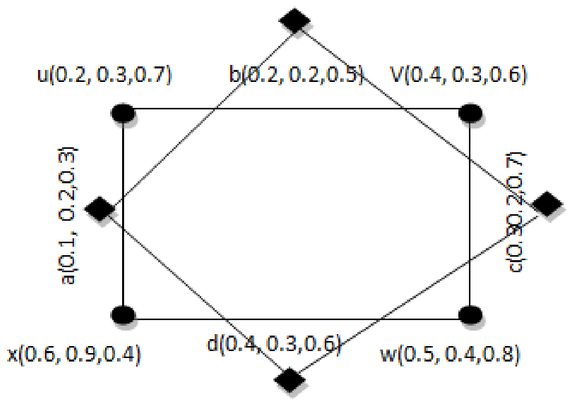
$= 0$ otherwise

$F_{B_{Q_1tl}}(e, f) = F_B(e) \vee F_B(f)$ if $e, f \in E$ & $t$hey have a vertex in common

$= 0$ otherwise

Thus by the definition B_{Q1tl} is a single valued neutrosophic relation on A_{Q1tl}. Hence the pair $Q_1tl(G) = (A_{Q_1tll}, B_{Q_1tl})$ is a SVN graph and is termed as 1 – Quasi Total Single Valued Neutrosophic Graph.





G **$Q_1tl(G)$**

In the above $Q_1tl(G)$, (u, v) = (0.2, 0.2, 0.5), (v, w) = (0.3, 0.2, 0.7), (w, x) = (0.4, 0.3, 0.6), (x, u) = (0.1,0.2, 0.3), (a, b) = (0.1, 0.2, 0.5), (b, c) = (0.2, 0.2, 0.7), (c, d) = (0.3, 0.2, 0.7),

(d, a) = (0.1, 0.2, 0.6)

3. 2 – Quasi Total SVNG

Definition 3.1

Let G=(A,B) be a SVN graph with its underlying crisp graph $G^* = (V, E)$. The pair $Q_2tl(G) = (A_{Q_2tl}, B_{Q_2tll})$ of G is defined as follows. The vertex set of $Q_2tl(G)$ is $V \cup E$. The SVN subset A_{Q2tl} is defined on $V \cup E$ as,

$(T_A, I_A, F_A)_{Q_2tl}(x) = (T_A, I_A, F_A)(x)$ if $x \in V$

$= (T_B, I_B, F_B)(x)$ if $x \in E$

The SVN relation B_{Q_1tl} on $V \cup E$ is defined as

$T_{B_{Q_2tl}}(x,y) = T_B(x,y), I_{B_{Q_2tll}}(x,y) = I_B(x,y), F_{B_{Q_2tl}}(x,y) = F_B(x,y)$ if $(x,y) \in E$

$T_{B_{Q_2tl}}(x,e) = T_B(x) \wedge T_B(e)$ if $x \in V, e \in E$ & x *lies on* e

= 0 otherwise

$I_{B_{Q_2tl}}(x,e) = I_B(x) \wedge I_B(e)$ if $x \in V, e \in E$ & x *lies on* e

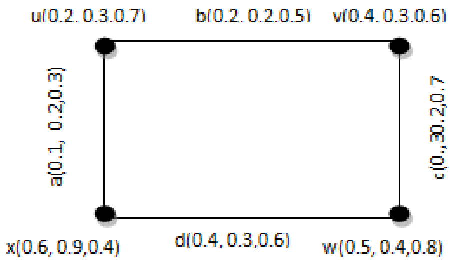
= 0 otherwise

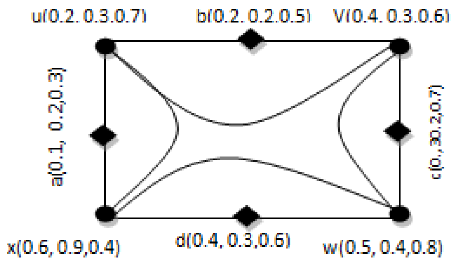
$F_{B_{Q_2tl}}(x,e) = F_B(e)$ if $x \in V, e \in E$ & x *lies on* e

= 0 otherwise

Thus by the definition B_{Q2tl} is a single valued neutrosophic relation on A_{Q2tl}. Hence the pair $Q_2tl(G) = (A_{Q_2tll}, B_{Q_2tl})$ is a SVN graph and is termed as 2 – Quasi Total Single Valued Neutrosophic Graph.

Example 3.2





G **Q_2tl(G)**

In the above Q_2tl(G),), (u, v) = (0.2, 0.2, 0.5), (v, w) = (0.3, 0.2, 0.7), (w, x) = (0.4, 0.3, 0.6), (x, u) = (0.1,0.2, 0.3), (a,u) =(0.1,0.2,0.3), (u,b) = (0.2,0.2,0.5), (b,v) = (0.2,0.2,0.5),

(v,c) = (0.3,0.2,0.7), (c,w) = (0.3,0.2,0.7), (w,d) = (0.4,0.3,0.6), (d,x) = (0.4,0.3,0.6),

(x,a) = (0.1,0.2,0.3)

Properties of 2 – Quasi Total SVN Graph

Theorem 3.3

Let G=(A,B) be SVN graph and Q_2 tl(G) is its 2 – Quasi Total SVN graph, order of Q_2tl(G) = order(G) + size(G).

Proof

By definition of Q_2tl(G), vertex set of Q_2tl(G) is V ∪ E.

Order of Q_2tl(G) = $\left(O_T(\,Q_2tl(G)), O_I(\,Q_2tl(G)), O_F(Q_2tl(G))\right)$

$= \left(\sum_{x\in V\cup E} T_{A_{Q_2tl}}(x), \sum_{x\in V\cup E} I_{A_{Q_2tl}}(x), \sum_{x\in V\cup E} F_{A_{Q_2tl}}(x)\right)$

$= \left(\sum_{x\in V} T_{A_{Q_2tl}}(x) + \sum_{x\in E} T_{A_{Q_2tl}}(x), \sum_{x\in V} I_{A_{Q_2tl}}(x) + \sum_{x\in E} I_{A_{Q_2tll}}(x), \sum_{x\in V} F_{A_{Q_2tl}}(x) + \sum_{x\in E} F_{A_{Q_2tl}}(x)\right)$ =

$$\left(\sum_{x\in V} T_{A_{Q_2tl}}(x), \sum_{x\in V} I_{A_{Q_2tl}}(x), \sum_{x\in V} F_{A_{Q_2tl}}(x)\right) + \left(\sum_{x\in E} T_{A_{Q_2tl}}(x), \sum_{x\in E} I_{A_{Q_2tl}}(x), \sum_{x\in E} F_{A_{Q_2tl}}(x)\right)$$

= order(G) + size (G).

Theorem 3.4

Let G=(A,B) be SVN graph and $Q_2tl(G)$ is its 2 - Quasi Total SVN graph, size of $Q_2tl(G)$ = 3 size(G)

Proof

size of $Q_2tl(G)$= $\left(S_T(Q_2tl(G)), S_I(Q_2tl(G)), S_F(Q_2tl(G))\right)$

$$= \left(\sum_{x,y\in V\cup E} T_{B_{Q_2tl}}(x,y), \sum_{x,y\in V\cup E} I_{B_{Q_2tl}}(x,y), \sum_{x,y\in V\cup E} F_{B_{Q_2tl}}(x,y)\right)$$

$$= \left(\left(\sum_{x,y\in V} T_{B_{Q_2tl}}(x,y), \sum_{x,y\in V} I_{B_{Q_2tl}}(x,y), \sum_{x,y\in V} F_{B_{Q_2tl}}(x,y)\right) + \left(\sum_{x\in V,y\in E} T_{B_{Q_2tl}}(x,y), \sum_{x\in V,y\in E} I_{B_{Q_2tl}}(x,y), \sum_{x\in V,y\in E} F_{B_{Q_2tl}}(x,y)\right)\right)$$

x lies on y in second paranthesis

$$= \left(\sum_{x,y\in V} T_{B_{Q_2tl}}(x,y), \sum_{x,y\in V} I_{B_{Q_2tl}}(x,y), \sum_{x,y\in V} F_{B_{Q_2tl}}(x,y)\right) + \left(\sum_{x\in V,y\in E} T_A(x)\wedge T_B(y), \sum_{x\in V,y\in E} I_A(x)\wedge I_B(y), \sum_{x\in V,y\in E} F_B(y)\right)$$

$= \text{size}(G) + (2\sum T_B(y), 2\sum I_B(y), 2\sum F_B(y))$

= size(G) + 2 size(G)

= 3 size (G)

REMARKS

- If (x, y) ∈ E(G), then there exist a triangle in $E(Q_2tl(G))$ containing (x, y) as one of the edges.
- If e = (x,y) is not in a triangle of G and e ∈ E(G) is only the edge between the vertices x and y in G, then there is only one triangle in $E(Q_2tl(G))$ containing (x, y) as one of the edges.
- Every triangle in $Q_2tl(G)$ contains an edge of G.
- If G is a graph containing only one edge then the graph $Q_2tl(G)$ contains unique triangle.

4. ISOMORPHIC RELATION BETWEEN SOME SPECIAL TYPES OF SVNG

Theorem 4.1

Let G be single valued neutrosophic graph, $Q_2tl(G)$ is weak isomorphic to tl(G).

Proof

Let G = (A,B) be a SVN graph with its underlying crisp graph G^*= (V,E). By the definition of $Q_2tl(G)$, A_{Q2tl} is a SVN subset defined on V ∪ E as

$$(T_A, I_A, F_A)_{Q2tl}(x) = (T_A, I_A, F_A)(x) \text{ if } x \in V$$

$= (T_B, I_B, F_B)(x)$ if $x \in E$ (1)

The SVN relation B_{Q2tl} on $V \cup E$ is defined as

$T_{B_{Q_2tl}}(x,y) = T_B(x,y), I_{B_{Q_2tll}}(x,y) = I_B(x,y), F_{B_{Q_2tl}}(x,y) = F_B(x,y)$ if $(x,y) \in E$

$T_{B_{Q_2tl}}(x,e) = T_B(x) \wedge T_B(e)$ if $x \in V, e \in E$ & x *lies on* e

= 0 otherwise

$I_{B_{Q_2tl}}(x,e) = I_B(x) \wedge I_B(e)$ if $x \in V, e \in E$ & x *lies on* e

= 0 otherwise

$F_{B_{Q_2tl}}(x,e) = F_B(e)$ if $x \in V, e \in E$ & x *lies on* e

= 0 otherwise

Using (1) in the above equation,

$T_{B_{Q_2tl}}(x,e) = T_{A_{Q_2tl}}(x) \wedge T_{A_{Q_2tl}}(e)$ if $x \in V, e \in E$ & x *lies on* e

= 0 otherwise

$I_{B_{Q_2tl}}(x,e) = I_{A_{Q_2tl}}(x) \wedge I_{A_{Q_2tl}}(e)$ if $x \in V, e \in E$ & x *lies on* e

= 0 otherwise

$F_{B_{Q_2tl}}(x,e) = F_{A_{Q_2tl}B}(e)$ if $x \in V, e \in E$ & x *lies on* e

= 0 otherwise

Define a map 'g' from $Q_2tl(G)$ to tl(G) as identity map $g: V \cup E \rightarrow V \cup E$, g be bijection satisfying

$(T_A, I_A, F_A)_{tl}(g(x)) = (T_A, I_A, F_A)_{tl}(x) = (T_A, I_A, F_A)(x) = (T_A, I_A, F_A)_{Q_2tl}(x)$ if $x \in V$

$(T_A, I_A, F_A)_{tl}(g(x)) = (T_A, I_A, F_A)_{tl}(x) = (T_B, I_B, F_B)(x) = (T_A, I_A, F_A)_{Q_2tl}(x)$ if $x \in E$

That is $(T_A, I_A, F_A)_{tl}(g(x)) = (T_A, I_A, F_A)_{Q_2tl}(x)$ if $x \in V \cup E$

Case 1

If $x, y \in V$, $(T_B, I_B, F_B)_{tl}\big(g(x), g(y)\big) = (T_B, I_B, F_B)_{tl}(x,y) = (T_B, I_B, F_B)(x,y)$ if $x, y \in V$.

By the definition of Q_2tl(G), $(T_B, I_B, F_B)_{Q_2tl}(x,y) = (T_B, I_B, F_B)(x,y)$ if $x, y \in V$ by the definition of Q_2tl(G)

That implies $(T_B, I_B, F_B)_{Q_2tl}(x,y) = (T_B, I_B, F_B)_{tl}(g(x), g(y))$ if $x, y \in V$

Case 2

If $x \in V$ and $y = e \in E$, then

$T_{B_{tl}}\big(g(x), g(e)\big) = T_{B_{tl}}(x,e) = \min\{T_A(x), T_B(e)\}$ if $x \in V, e \in E$ and x lies on e

= 0 otherwise

$I_{B_{tl}}(g(x), g(e)) = I_{B_{tl}}(x, e) = \min\{I_A(x), I_B(e)\}$ if $x \in V, e \in E$ and x lies on e

$= 0$ otherwise

$F_{B_{tl}}(g(x), g(e)) = F_{B_{tl}}(x, e) = F_B(e)$ if $x \in V, e \in E$ and x lies on e

$= 0$ otherwise

$T_{B_{Q_2tl}}(x, e) = T_A(x) \wedge T_B(e)$ if $x \in V, e \in E$ & *x lies on e*

$= 0$ otherwise

$I_{B_{Q_2tl}}(x, e) = I_A(x) \wedge I_B(e)$ if $x \in V, e \in E$ & *x lies on e*

$= 0$ otherwise

$F_{B_{Q_2tl}}(x, e) = F_B(e)$ if $x \in V, e \in E$ & *x lies on e*

$= 0$ otherwise

That implies $(T_B, I_B, F_B)_{Q_2tl}(x, y) = (T_B, I_B, F_B)_{tl}(g(x), g(y))$

Case 3

If $x = e_i, y = e_j \in E$ then

$T_{B_{tl}}(e_i, e_j) = \min\{T_B(e_i), T_B(e_j)\}$ if e_i, e_j have a vertex in common

$I_{B_{tl}}(e_i, e_j) = \min\{I_B(e_i), I_B(e_j)\}$ if e_i, e_j have a vertex in common

$F_{B_{tl}}(e_i, e_j) = \max\{F_B(e_i), F_B(e_j)\}$ if e_i, e_j have a vertex in common

$= 0$ otherwise

$(T_B, I_B, F_B)_{Q_2tl}(e_i, e_j) = 0$

$(T_B, I_B, F_B)_{Q_2tl}(x, y) \le (T_B, I_B, F_B)_{tl}(g(x), g(y))$

Thus from the above cases we get $T_{B_{Q_2tl}}(x, y) \le T_{B_{tl}}(x, y)$ if $x, y \in V \cup E$

$I_{B_{Q_2tl}}(x, y) \le I_{B_{tl}}(x, y)$ if $x, y \in V \cup E$

$F_{B_{Q_2tl}}(x, y) \le F_{B_{tl}}(x, y)$ if $x, y \in V \cup E$

Therefore $g: Q_2tl(G) \to tl(G)$ is a weak isomorphism.

Theorem 4.2

Let G be single valued neutrosophic graph, $Q_2tl(G)$ is isomorphic to $G \cup sd(G)$.

Proof

Let G = (A, B) be a SVN graph with the underlying crisp graph $G^* = (V, E)$. Let $H = G \cup sd(G)$; H = (P,Q). H has its node as $V \cup E$. By definition

$T_P(x) = T_A(x)$ if $x \in V$

$= T_B(x)$ if $x \in E$

$I_P(x) = I_A(x)$ if $x \in V$

$= I_B(x)$ if $x \in E$

$F_P(x) = F_A(x)$ if $x \in V$

$= F_B(x)$ if $x \in E$

$T_Q(x, y) = T_B(x, y)$ if $(x, y) \in E$

$= T_{B_{sd}}(x, y)$ if $(x, y) \in E(sd(G))$

$I_Q(x, y) = I_B(x, y)$ if $(x, y) \in E$

$= I_{B_{sd}}(x, y)$ if $(x, y) \in E(sd(G))$

$F_Q(x, y) = F_B(x, y)$ if $(x, y) \in E$

$= F_{B_{sd}}(x, y)$ if $(x, y) \in E(sd(G))$

Let us consider the identity map $k : Q_2tl(G) \to G \cup sd(G)$ by the definition of $Q_2tl\ (G)$

$T_{A_{Q_2tl}}(x) = T_A(x)$ if $x \in V$

$= T_B(x)$ if $x \in E$

$I_{A_{Q_2tl}}(x) = I_A(x)$ if $x \in V$

$= I_B(x)$ if $x \in E$

$F_{A_{Q_2tl}}(x) = F_A(x)$ if $x \in V$

$= F_B(x)$ if $x \in E$

$T_P(k(x)) = T_P(x) = T_A(x)$ if $x \in V$

$= T_B(x)$ if $x \in E$

$I_P(k(x)) = I_P(x) = I_A(x)$ if $x \in V$

$= I_B(x)$ if $x \in E$

$F_P(k(x)) = F_P(x) = F_A(x)$ if $x \in V$

$= F_B(x)$ if $x \in E$

So, $T_{A_{Q_2tl}}(x) = T_P(k(x))$ if $x \in V \cup E$

$I_{A_{Q_2tl}}(x) = I_P(k(x))$ if $x \in V \cup E$

$F_{A_{Q_2tl}}(x) = F_P(k(x))$ if $x \in V \cup E$ (2)

Case – 1

If $x, y \in V$ and $(x, y) \in E$ then

$$T_{B_{Q_2tl}}(x, y) = T_B(x, y)$$

$$I_{B_{Q_2tl}}(x, y) = I_B(x, y)$$

$$F_{B_{Q_2tl}}(x, y) = F_B(x, y)$$

$$T_Q(k(x), k(y)) = T_Q(x, y) = T_B(x, y)$$

$$I_Q(k(x), k(y)) = I_Q(x, y) = I_B(x, y)$$

$$F_Q(k(x), k(y)) = F_Q(x, y) = F_B(x, y)$$

Therefore $T_{B_{Q_2tl}}(x, y) = T_Q(k(x), k(y))$

$$I_{B_{Q_2tl}}(x, y) = I_Q\big(k(x), k(y)\big)$$

$$F_{B_{Q_2tl}}(x, y) = F_Q(k(x), k(y))$$

Case – 2

If $x \in V, e \in E$ and x lies on e

$$T_{B_{Q_2tl}}(x, e) = \min\{T_A(x), T_B(e)\}$$

$$I_{B_{Q_2tl}}(x, y) = \min\{I_A(x), I_B(e)\}$$

$$F_{B_{Q_2tl}}(x, y) = F_B(e)$$

$$T_Q(k(x), k(e)) = T_{B_{sd}}(x, e) = \min\{T_A(x), T_B(e)\}$$

$$I_Q(k(x), k(e)) = I_{B_{sd}}(x, e) = \min\{I_A(x), I_B(e)\}$$

$$F_Q(k(x), k(e)) = F_{B_{sd}}(x, e) = F_B(e)$$

Therefore $T_{B_{Q_2tl}}(x, e) = T_Q(k(x), k(e))$

$$I_{B_{Q_2tl}}(x, e) = I_Q\big(k(x), k(e)\big)$$

$$F_{B_{Q_2tl}}(x, e) = F_Q(k(x), k(e))$$

Hence by the above two cases and by the definition of H

$$(T, I, F)_{Q_2tl}(x, y) = (T, I, F)_Q(k(x), k(y)) \text{ if } (x, y) \in E \tag{3}$$

$$(T, I, F)_{Q_2tl}(x, e) = (T, I, F)_{sd}(x, e) = (T, I, F)_Q(k(x), k(e)) \text{ if } x \in V, e \in E \text{ and x lies on e} \tag{4}$$

The identity map $k : tl(G) \to G \cup sd(G)$ is bijective and from (2), (3) and (4) $Q_2tl(G)$ is isomorphic with $G \cup sd(G)$.

Theorem 4.3

G be single valued neutrosophic graph, tl(G) is isomorphic to $M(G) \cup Q_2\text{tl}(G)$.

Proof

Let G = (A, B) be a SVN graph with the underlying crisp graph $G^* = (V, E)$. Let $H = M(G) \cup Q_2\text{tl}(G)$; H = (P,Q). H has its node as V ∪ E. By definition

$T_P(x) = T_A(x)$ if $x \in V$

$= T_B(x)$ if $x \in E$

$I_P(x) = I_A(x)$ if $x \in V$

$= I_B(x)$ if $x \in E$

$F_P(x) = F_A(x)$ if $x \in V$

$= F_B(x)$ if $x \in E$

$T_Q(x, y) = T_{B_M}(x, y)$ if $(x, y) \in E(sd(G))$

$= T_{B_{Q_2\text{tl}}}(x, y)$ if $(x, y) \in E(Q_2\text{tl}(G))$

$I_Q(x, y) = I_{B_M}(x, y)$ if $(x, y) \in E(sd(G))$

$= I_{B_{Q_2\text{tl}}}(x, y)$ if $(x, y) \in E(Q_2\text{tl}(G))$

$F_Q(x, y) = F_{B_M}(x, y)$ if $(x, y) \in E(sd(G))$

$= F_{B_{Q_2\text{tl}}}(x, y)$ if $(x, y) \in E(Q_2\text{tl}(G))$

Let us consider the identity map $f : \text{tl}(G) \to M(G) \cup Q_2\text{tl}(G)$ by the definition of tl(G)

$T_{A_{tl}}(x) = T_A(x)$ if $x \in V$

$= T_B(x)$ if $x \in E$

$I_{A_{tl}}(x) = I_A(x)$ if $x \in V$

$= I_B(x)$ if $x \in E$

$F_{A_{tl}}(x) = F_A(x)$ if $x \in V$

$= F_B(x)$ if $x \in E$

$T_P(f(x)) = T_P(x) = T_A(x)$ if $x \in V$

$= T_B(x)$ if $x \in E$

$I_P(f(x)) = I_P(x) = I_A(x)$ if $x \in V$

$= I_B(x)$ if $x \in E$

$F_P(f(x)) = F_P(x) = F_A(x)$ if $x \in V$

$= F_B(x)$ if $x \in E$

So, $T_{A_{tl}}(x) = T_P(f(x))$ if $x \in V \cup E$

$I_{A_{tl}}(x) = I_P(f(x))$ if $x \in V \cup E$

$F_{A_{tl}}(x) = F_P(f(x))$ if $x \in V \cup E$ (5)

Case – 1:

If $x, y \in V$ and $(x, y) \in E$ then

$T_{B_{tl}}(x, y) = T_B(x, y)$

$I_{B_{tl}}(x, y) = I_B(x, y)$

$F_{B_{tl}}(x, y) = F_B(x, y)$

$T_Q(f(x), f(y)) = T_Q(x, y) = T_B(x, y)$

$I_Q(f(x), f(y)) = I_Q(x, y) = I_B(x, y)$

$F_Q(f(x), f(y)) = F_Q(x, y) = F_B(x, y)$

Therefore $T_{B_{tl}}(x, y) = T_Q(f(x), f(y))$

$I_{B_{tl}}(x, y) = I_Q\big(f(x), f(y)\big)$

$F_{B_{tl}}(x, y) = F_Q(f(x), f(y))$

Case – 2:

If $x \in V$ and $y \in E$ x lies on y

$T_{B_{tl}}(x, y) = \min\{T_A(x), T_B(y)\}$

$I_{B_{tl}}(x, y) = \min\{I_A(x), I_B(y)\}$

$F_{B_{tl}}(x, y) = F_B(y)$

$T_Q(f(x), f(y)) = T_{B_{Q_2 tl}}(x, y) = \min\{T_A(x), T_B(y)\}$

$I_Q(f(x), f(y)) = I_{B_{Q_2 tl}}(x, y) = \min\{I_A(x), I_B(y)\}$

$F_Q(f(x), f(y)) = F_{B_{Q_2 tl}}(x, y) = F_B(y)$

Therefore $T_{B_{tl}}(x, y) = T_Q(f(x), f(y))$

$I_{B_{tl}}(x, y) = I_Q\big(f(x), f(y)\big)$

$F_{B_{tl}}(x, y) = F_Q(f(x), f(y))$

Case – 3

If $x, y \in E$ and are adjacent in G^*

$$T_{B_{tl}}(x,y) = \min\{T_B(x), T_B(y)\}$$

$$I_{B_{tl}}(x,y) = \min\{I_B(x), I_B(y)\}$$

$$F_{B_{tl}}(x,y) = \max\{F_B(x), F_B(y)\}$$

$$T_Q(f(x), f(y)) = T_{B_M}(x,y) = \min\{T_B(x), T_B(y)\}$$

$$I_Q(f(x), f(y)) = I_{B_M}(x,y) = \min\{I_B(x), I_B(y)\}$$

$$F_Q(f(x), f(y)) = F_{B_M}(x,y) = \max\{F_B(x), F_B(y)\}$$

Therefore $T_{B_{tl}}(x,y) = T_Q(f(x), f(y))$

$$I_{B_{tl}}(x,y) = I_Q\big(f(x), f(y)\big)$$

$$F_{B_{tl}}(x,y) = F_Q(f(x), f(y))$$

Hence by the above three cases and by the definition of H

$$(T,I,F)_{B_{tl}}(x,y) = (T,I,F)_Q(f(x), f(y)) \text{ if } (x,y) \in E \tag{6}$$

$$(T,I,F)_{B_{tl}}(x,e) = (T,I,F)_{Q_2\mathrm{tl}}(x,e) = (T,I,F)_Q(k(x), k(e)) \text{ if } x \in V \text{ and } e \in E \tag{7}$$

$$(T,I,F)_{B_{tl}}(e,f) = (T,I,F)_M(e,f) = (T,I,F)_Q(k(e), k(f)) \text{ if } e, f \in E \tag{8}$$

The identity map $k : \mathrm{tl}(G) \to M(G) \cup Q_2\mathrm{tl}(G)$ is bijective and from (5), (6), (7) and (8) tl(G) is isomorphic with $M(G) \cup Q_2\mathrm{tl}(G)$.

Theorem 4.4

G be single valued neutrosophic graph, tl(G) is isomorphic to $Q_1\mathrm{tl}(G) \cup Q_2\mathrm{tl}(G)$.

Proof

Let G = (A, B) be a SVN graph with the underlying crisp graph $G^* = (V, E)$. Let $H = Q_1\mathrm{tl}(G) \cup Q_2\mathrm{tl}(G)$; H = (P,Q). H has its node as $V \cup E$. By definition

$$T_P(x) = T_A(x) \text{ if } x \in V$$

$$= T_B(x) \text{ if } x \in E$$

$$I_P(x) = I_A(x) \text{ if } x \in V$$

$$= I_B(x) \text{ if } x \in E$$

$$F_P(x) = F_A(x) \text{ if } x \in V$$

$$= F_B(x) \text{ if } x \in E$$

$$T_Q(x,y) = T_{B_{Q_1\mathrm{tl}}}(x,y) \text{ if } (x,y) \in E(Q_1\mathrm{tl}(G))$$

$= T_{B_{Q_2 tl}}(x,y)$ if $(x,y) \in E(Q_2 tl(G))$

$I_Q(x,y) = I_{B_{Q_1 tl}}(x,y)$ if $(x,y) \in E(Q_1 tl(G))$

$= I_{B_{Q_2 tl}}(x,y)$ if $(x,y) \in E(Q_2 tl(G))$

$F_Q(x,y) = F_{B_{Q_1 tl}}(x,y)$ if $(x,y) \in E(Q_1 tl(G))$

$= F_{B_{Q_2 tl}}(x,y)$ if $(x,y) \in E(Q_2 tl(G))$

Let us consider the identity map $h : tl(G) \to Q_1 tl(G) \cup Q_2 tl(G)$ by the definition of tl(G)

$T_{A_{tl}}(x) = T_A(x)$ if $x \in V$

$= T_B(x)$ if $x \in E$

$I_{A_{tl}}(x) = I_A(x)$ if $x \in V$

$= I_B(x)$ if $x \in E$

$F_{A_{tl}}(x) = F_A(x)$ if $x \in V$

$= F_B(x)$ if $x \in E$

$T_P(h(x)) = T_P(x) = T_A(x)$ if $x \in V$

$= T_B(x)$ if $x \in E$

$I_P(h(x)) = I_P(x) = I_A(x)$ if $x \in V$

$= I_B(x)$ if $x \in E$

$F_P(h(x)) = F_P(x) = F_A(x)$ if $x \in V$

$= F_B(x)$ if $x \in E$

So, $T_{A_{tl}}(x) = T_P(h(x))$ if $x \in V \cup E$

$I_{A_{tl}}(x) = I_P(h(x))$ if $x \in V \cup E$

$F_{A_{tl}}(x) = F_P(h(x))$ if $x \in V \cup E$ (9)

Case – 1

If $x, y \in V$ and $(x,y) \in E$ then

$T_{B_{tl}}(x,y) = T_B(x,y)$

$I_{B_{tl}}(x,y) = I_B(x,y)$

$F_{B_{tl}}(x,y) = F_B(x,y)$

$T_Q(h(x), h(y)) = T_Q(x,y) = T_B(x,y)$

$I_Q(h(x), h(y)) = I_Q(x, y) = I_B(x, y)$

$F_Q(h(x), h(y)) = F_Q(x, y) = F_B(x, y)$

Therefore $T_{B_{tl}}(x, y) = T_Q(h(x), h(y))$

$I_{B_{tl}}(x, y) = I_Q\big(h(x), h(y)\big)$

$F_{B_{tl}}(x, y) = F_Q(h(x), h(y))$

Case – 2

If x ∈ V and y ∈ E x lies on y

$T_{B_{tl}}(x, y) = \min\{T_A(x), T_B(y)\}$

$I_{B_{tl}}(x, y) = \min\{I_A(x), I_B(y)\}$

$F_{B_{tl}}(x, y) = F_B(y)$

$T_Q(h(x), h(y)) = T_{B_{Q_2 tl}}(x, y) = \min\{T_A(x), T_B(y)\}$

$I_Q(h(x), h(y)) = I_{B_{Q_2 tl}}(x, y) = \min\{I_A(x), I_B(y)\}$

$F_Q(h(x), h(y)) = F_{B_{Q_2 tl}}(x, y) = F_B(y)$

Therefore $T_{B_{tl}}(x, y) = T_Q(h(x), h(y))$

$I_{B_{tl}}(x, y) = I_Q\big(h(x), h(y)\big)$

$F_{B_{tl}}(x, y) = F_Q(h(x), h(y))$

Case – 3

If x, y ∈ E and are adjacent in G^*

$T_{B_{tl}}(x, y) = \min\{T_B(x), T_B(y)\}$

$I_{B_{tl}}(x, y) = \min\{I_B(x), I_B(y)\}$

$F_{B_{tl}}(x, y) = \max\{F_B(x), F_B(y)\}$

$T_Q(h(x), h(y)) = T_{B_{Q_I tl}}(x, y) = \min\{T_B(x), T_B(y)\}$

$I_Q(h(x), h(y)) = I_{B_{Q_I tl}}(x, y) = \min\{I_B(x), I_B(y)\}$

$F_Q(h(x), h(y)) = F_{B_{Q_I tl}}(x, y) = \max\{F_B(x), F_B(y)\}$

Therefore $T_{B_{tl}}(x, y) = T_Q(h(x), h(y))$

$I_{B_{tl}}(x, y) = I_Q\big(h(x), h(y)\big)$

$F_{B_{tl}}(x, y) = F_Q(h(x), h(y))$

Hence by the above three cases and by the definition of H

$$(T, I, F)_{B_{tl}}(x, y) = (T, I, F)_{Q}(h(x), h(y)) \text{ if } (x, y) \in E \tag{10}$$

$$(T, I, F)_{B_{tl}}(x, y) = (T, I, F)_{Q_2 tl}(x, y) = (T, I, F)_{Q}(h(x), h(y)) \text{ if } x \in V \text{ and } e \in E \tag{11}$$

$$(T, I, F)_{B_{tl}}(x, y) = (T, I, F)_{Q_1 tl}(x, y) = (T, I, F)_{Q}(h(x), h(y)) \text{ if } e, f \in E \tag{12}$$

The identity map $k : tl(G) \to Q_1 tl(G) \cup Q_2 tl(G)$.is bijective and from (9), (10), (11) and (12) tl(G) is isomorphic with $Q_1 tl(G) \cup Q_2 tl(G)$.

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